5.2 - Sum and Difference Identities

Warmup

Find the exact value

1) $\cos(45° - 30°)$

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

2) $\cos(120° - 45°)$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

3) $\sin(45° - 30°)$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

4) $\sin(60° - 45°)$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

5.1 - Trigonometric Identities

Practice - Find the missing term

1) $\dfrac{\cos x}{1 - \tan x} + \dfrac{\sin x}{1 - \cot x} = \sin x + ???$ $??? = \cos x$

2) $\csc x - \cot x = \dfrac{???}{1 + \cos}$ $??? = \sin x$

Chapter 5
Trigonometric Identities

5.4 - Half-Angle Identities

Deriving the Half Angle Identities

$$\sin\left(\frac{\theta}{2}\right) \quad \text{and} \quad \cos\left(\frac{\theta}{2}\right)$$

Start with the Cosine double-angle identity

$$\cos(2A) = 1 - 2\sin^2 A$$

$$2\sin^2 A = 1 - \cos(2A)$$

$$\sin^2 A = \frac{1 - \cos(2A)}{2}$$

$$\sin A = \pm\sqrt{\frac{1 - \cos(2A)}{2}} \quad \text{Substitute } A = \frac{\theta}{2}$$

$$\boxed{\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{2}}}$$

5.4 - Half-Angle Identities

Deriving the Half Angle Identities

$$\sin\left(\frac{\theta}{2}\right) \quad \text{and} \quad \cos\left(\frac{\theta}{2}\right)$$

Start with the other Cosine double-angle identity

$$\cos(2A) = 2\cos^2 A - 1$$

$$2\cos^2 A = \cos(2A) + 1$$

$$\cos^2 A = \frac{1 + \cos(2A)}{2}$$

$$\cos A = \pm\sqrt{\frac{1 + \cos(2A)}{2}} \quad \text{Substitute } A = \frac{\theta}{2}$$

$$\boxed{\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos\theta}{2}}}$$

5.4 - Half-Angle Identities

Deriving the Half Angle Identities

Start with the Tangent identity

$$\tan\left(\frac{A}{2}\right) = \frac{\sin\left(\frac{A}{2}\right)}{\cos\left(\frac{A}{2}\right)}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\pm\sqrt{\frac{1-\cos A}{2}}}{\pm\sqrt{\frac{1+\cos A}{2}}}$$

$$\boxed{\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}}$$

5.4 - Half-Angle Identities

Deriving the Half Angle Identities

Another way to derive a Tangent identity

$$1 - \cos A = 2 \sin^2 \left(\frac{A}{2} \right) \quad \text{and} \quad \sin A = 2 \sin \left(\frac{A}{2} \right) \cos \left(\frac{A}{2} \right)$$

$$\boxed{\frac{1 - \cos A}{\sin A}} = \frac{2 \sin^2 \frac{A}{2}}{2 \sin \frac{A}{2} \cos \frac{A}{2}} = \frac{\sin \frac{A}{2}}{\cos \frac{A}{2}} \boxed{= \tan \frac{A}{2}}$$

$$\boxed{\tan \left(\frac{A}{2} \right) = \frac{1 - \cos A}{\sin A}}$$

5.4 - Half-Angle Identities

Deriving the Half Angle Identities

Yet another way to derive a Tangent identity

$$1 + \cos A = 2\cos^2\left(\frac{A}{2}\right) \quad \text{and} \quad \sin A = 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$$

$$\frac{\sin A}{1 + \cos A} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{2\cos^2\frac{A}{2}} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = \tan\frac{A}{2}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1 + \cos A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1 - \cos A}{\sin A}$$

5.4 - Half-Angle Identities

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1 - \cos A}{\sin A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1 + \cos A}$$

Practice - Use the half-angle identities

1. $\cos\left(\dfrac{5\pi}{8}\right)$

2. $\tan\left(\dfrac{11\pi}{12}\right)$

3. $\cos x = \dfrac{1}{5}, \; find \cos\left(\dfrac{x}{2}\right)$

$-\dfrac{\sqrt{2 - \sqrt{2}}}{2}$

$-2 + \sqrt{3}$ or $-\sqrt{7 - 4\sqrt{3}}$

$\pm\dfrac{\sqrt{15}}{5}$

5.4 - Half-Angle Identities

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1 - \cos A}{\sin A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1 + \cos A}$$

Practice - Find the exact value, x is in 1st quadrant

$$\cos(x) = \frac{5}{13}$$

1. $\cos\left(\dfrac{x}{2}\right)$

$$\frac{3\sqrt{13}}{13}$$

2. $\sin\left(\dfrac{x}{4}\right)$

$$\frac{\sqrt{338 - 78\sqrt{13}}}{26}$$

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